ABRHS PHYSICS (CP) NAME: KEY

Centripetal Acceleration

Concepts

A. If you are going in a circle with a constant speed, why are you accelerating?

B. If you are going in a circle with a constant speed, in what direction do you accelerate?

Always to the center of the circle. The acceleration is also always ⊥ to your velocity.

C. If you are going in a circle with a constant speed, describe the direction of your velocity.

It is always tangent to the circle.

Calculations

1. A car is traveling in a circle with a radius of 20 meters.
   a. If it has a speed of 5 m/s, what is the acceleration of the car?

$r = 20\text{ m}$, $v = 5\text{ m/s}$ $\quad a_c = \frac{v^2}{r}$ $\quad a_c = \frac{(5)^2}{20} = \frac{25}{20} = 1.25\text{ m/s}^2$

   b. If it has a speed of 10 m/s, what is its acceleration?

$v = 10\text{ m/s}$ $\quad a_c = \frac{v^2}{r} = \frac{(10)^2}{20} = \frac{100}{20} = 5\text{ m/s}^2$

2. A plane is flying at 125 m/s when it begins to travel in a circle. If its centripetal acceleration is 2 m/s^2, what is the radius of the circle?

$a_c = 2\text{ m/s}^2$, $v = 125\text{ m/s}$ $\quad a_c = \frac{v^2}{r}$ $\quad 2 = \frac{(125)^2}{r}$ $\quad r = \frac{(125)^2}{2}$ $\quad r = 7813\text{ m}$

3. A girl is sitting on a merry-go-round 2 meters from the center.
   a. If she has an acceleration of 1 m/s^2, how fast is she going?

$r = 2\text{ m}$, $a = 1\text{ m/s}^2$ $\quad a_c = \frac{v^2}{r}$ $\quad 1 = \frac{v^2}{(2)}$ $\quad v^2 = 2$ $\quad v = 1.41\text{ m/s}$

   b. If she has an acceleration of 2 m/s^2, how fast is she going?

$a = 2\text{ m/s}^2$ $\quad a_c = \frac{v^2}{r}$ $\quad 2 = \frac{v^2}{2}$ $\quad v^2 = 4$ $\quad v = 2\text{ m/s}$

4. A person is driving in a circle with a centripetal acceleration of 2 m/s^2.
   a. What would be the acceleration if they went twice as fast, but kept the radius the same?

Since $a_c = \frac{v^2}{r}$ if we double the speed, we have

$$\frac{(2v)^2}{r} = \frac{4v^2}{r}$$

So 4 x the acceleration,

So $(4)(2) = 8\text{ m/s}^2$

Centripetal Acceleration

b. What would be the acceleration if they went three times as fast, but kept the radius the same?

$\frac{(3v)^2}{r} = 9\frac{v^2}{r} = (9)(2) = 18\ m/s^2$

c. What would be the acceleration if they doubled the radius, but kept their speed the same?

$\frac{v^2}{2r} = \frac{1}{2}\frac{v^2}{r} = \frac{1}{2}(2) = 1\ m/s^2$

d. What would be the acceleration if they tripled the radius, but kept their speed the same?

$\frac{v^2}{3r} = \frac{1}{3}\frac{v^2}{r} = \frac{1}{3}(2) = \frac{2}{3}\ m/s^2$

5. A car is traveling in a circle of radius 15 meters. It takes 9 seconds to go once around the circle. What is the centripetal acceleration? *(Hint: Find the speed first.)*

$r = 15\ m$
$T = 9\ s$
$a_c = ?$

$v = \frac{2\pi r}{T}$

$v = \frac{2\pi(15)}{9}$

$v = 10.47\ m/s$

$a_c = \frac{v^2}{r}$

$a_c = \frac{(10.47)^2}{15}$

$a_c = 7.3\ m/s^2$

6. A ball is swung on a string in a circle of radius 1.3 meters. If the centripetal acceleration of the ball is 15 m/s^2, how long does it take the ball to go around once? *(Hint: Find the speed first.)*

$r = 1.3\ m$
$a_c = 15\ m/s^2$
$T = ?$

$a_c = \frac{v^2}{r}$

$15 = \frac{v^2}{1.3}$

$v^2 = 19.5 \rightarrow v = 4.42\ m/s$

$v = \frac{2\pi r}{T}$

$4.42 = \frac{2\pi(1.3)}{T}$

$T = \frac{2\pi(1.3)}{(4.42)} = 1.85\ s$

7. While flying in circles, a plane has a centripetal acceleration of 5 m/s^2. If the radius of the turn is 8000 meters, how many seconds does it take to go around once? *(No more hints!)*

$a_c = 5\ m/s^2$
$r = 8000\ m$
$T = ?$

$a_c = \frac{v^2}{r}$

$5 = \frac{v^2}{8000}$

$v^2 = 40{,}000 \rightarrow v = 200\ m/s$

$v = \frac{2\pi r}{T}$

$200 = \frac{2\pi(8000)}{T}$

$T = \frac{2\pi(8000)}{200} = 251\ s$

8. A person is spinning on the Turkish Twist, which has a radius of 5 meters. If it takes 2.5 seconds to go around once, what is the centripetal acceleration of the person?

$r = 5\ m$
$T = 2.5\ s$
$a_c = ?$

$v = \frac{2\pi r}{T}$

$v = \frac{2\pi(5)}{2.5}$

$v = 12.6\ m/s$

$a_c = \frac{v^2}{r}$

$a_c = \frac{(12.6)^2}{5}$

$a_c = 31.6\ m/s^2$

* SEE BELOW FOR ANOTHER WAY TO DO 9 & 10 *

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9. A ball on the end of a string is being spun in a circle of radius 2.3 meters. It is spinning at a rate of 45 rpm. What is the centripetal acceleration of the ball?

$r = 2.3\text{ m}$

$f = 45\text{ rpm}$

$a_c = ?$

Notice: $a_c = \frac{v^2}{r}$ we know "r" so need to find v. Then $v = \frac{2\pi r}{T}$ so we need T!

$T = \frac{1}{f} = \frac{1}{45}$ $T = 0.0222\text{ min}$

$(0.0222\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right) = 1.333\text{ sec}$

$v = \frac{2\pi r}{T} = \frac{2\pi(2.3)}{1.33} = 10.83\text{ m/s}$

& $a_c = \frac{v^2}{r} = \frac{(10.83)^2}{2.3}$

$a_c = 51.1\text{ m/s}^2$

10. A person on a 10 meter radius Ferris wheel is rotating with a centripetal acceleration of 4 m/s^2. What is the rate of rotation in rpm?

$r = 10\text{ m}$

$a_c = 4\text{ m/s}^2$

$f = ?\text{ rpm}$

① $a_c = \frac{v^2}{r}$ $4 = \frac{v^2}{10}$ $v^2 = 40$ $v = 6.32\text{ m/s}$

② $v = \frac{2\pi r}{T}$ $6.32 = \frac{2\pi(10)}{T}$ $T = 9.94\text{ s}$

③ $T = (9.94\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right)$ $T = 0.166\text{ min}$

④ $f = \frac{1}{T} = \frac{1}{0.166}$ $f = 6.04\text{ rpm}$

Answers:

1. a) 1.25 m/s² b) 5 m/s² 2) 7800 m 3. a) 1.4 m/s b) 2 m/s

4. a) 8 m/s² b) 18 m/s² c) 1 m/s² d) 0.67 m/s²

5) v = 10.5 m/s & a = 7.3 m/s² 6) v = 4.42 m/s & t = 1.85 s 7) v = 200 m/s & t = 251 s

8) v = 12.6 m/s & a = 31.6 m/s² 9) v = 10.8 m/s & a = 51 m/s² 10) v = 6.32 m/s & T = 9.93 s & f = 6.04 rpm

You can think of "rpm" as just a different way of describing the velocity. The equations need to use "m/s" but people often use "rpm" to describe rotating things. To convert, just ask yourself how many seconds are in 1 minute and how many meters are in 1 rotation? [Answers: 60 seconds & $2\pi r$ meters]

Dimensional Analysis – like in chemistry ☺

9

$r = 2.3$ m

$f = 45$ rpm

$a_c = ?$

① $\left(45 \frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi(2.3) \text{ m}}{1 \text{ rev}}\right)$

So $V = 10.8$ m/s

② $a_c = \frac{v^2}{r}$

$a_c = \frac{(10.8)^2}{(2.3)}$

$a_c = 51 \text{ m/s}^2$

10

$r = 10$ m

$a_c = 4 \text{ m/s}^2$

rpm = ?

① $a_c = \frac{v^2}{r}$

$4 = \frac{v^2}{10}$

$v^2 = 40$

$v = 6.33$ m/s

② $\left(\frac{6.33 \text{ m}}{\text{s}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right)\left(\frac{1 \text{ rev}}{2\pi(10) \text{ m}}\right)$

$r = 6.04$ rpm